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APPLICATION OF FINISHED ELEMENTS FOR AN ANALYSIS OF CONCRETE AND ROCKS DISINTEGRATION PROCESSES BY ELECTROHYDRAULIC METHOD

ZASTOSOWANIE METODY ELEMENTÓW SKOŃCZONYCH W ANALIZIE PROCESU ROZPAJANIA BETONU I SKAŁ METODĄ ELEKTROHYDRAULICZNĄ

In the publication there was presented and discussed an algorithm of application of the finite elements method for an analysis of concrete and rocks state while undergoing disintegration processes by the use of electrohydraulic phenomenon. The description of phenomena taking place during the disintegration process of concrete blocks modelled in MSC Marc/Mentat pack was introduced with several simplifications, of which the most important was assumption of isotropic structure of the disintegrated material and absence of pressure losses in the process of high-voltage discharge (Bagci & Mamurekli, 2009).

A general distribution of stresses in the analysed blocks was presented and there was simulated a concrete blocks disintegration resulting from an increase of pressure generated in the shot hole. The analysis also allowed assessment of minimal values of pressures essential for disintegration of the examined blocks. Theoretical studies were confirmed through an experimental method by implementation of several disintegration tests of concrete blocks of differentiated strength parameters, sizes and concrete curing periods.

Keywords: electrohydraulic method, finite element method, splitting, concrete, rocks

W publikacji omówiono i przedstawiono algorytm wykorzystania metody elementów skończonych do analizy stanu betonu i skał poddawanych rozpajaniu na drodze wykorzystania do tego celu zjawiska elektrohydraulicznego.Opis zjawisk zachodzących podczas rozpajania bloków betonowych zamodelowanych w pakiecie MSC Marc/Mentat przedstawiony został przy przyjęciu szeregu uproszczeń, z których najistotniejszym było założenie izotropowej struktury rozspajanego materiału oraz brak strat ciśnienia w procesie wyładowania wysokonapięciowego. Przedstawiono uogólniony rozkład naprężeń w analizowanych blokach betonowych , oraz zasymulowano rozpajanie bloków betonowych na skutek wzrostu ciśnienia generowanego w otworze strzałowym. Analiza pozwoliła również na ustalenie minimalnych wartości ciśnień niezbędnych do rozpajania badanych bloków. Rozważańa teoretyczne potwierdzone zostały na drodze eksperymentalnej poprzez wykonanie szeregu prób rozpajanie bloków betonu o zróżnicowanych parametrach wytrzymałościowych, oraz zróżnicowanych gabarytach i okresie dojrzewania.

Słowa kluczowe: metoda elektrohydrauliczna, metoda elementów skończonych, rozpajanie, beton, skały

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1. Introduction

Numerical methods allow approximate solution of partial differential equations describing developed physical models, well representing real physical phenomena in conditions of e.g. occurrence of anisotropy, nonlinearity and changes of material constants values. They base of transformation of partial differential equations into algebraic equations computer solved with the use of proper software. Presently dominating computer calculation techniques, helping to solve highly complex systems of equations, allow at the same time to limit significantly calculation duration ensuring the required accuracy. Since the computers of high calculation potential got introduced, the method of finite elements (MES) has become the most popular one. It possesses applications for many areas of technique and is equipped with universal computer calculation procedures. An important advantage of the method, which facilitates an analysis of the obtained results, is visualization of achieved solutions in three-dimensional space where they can be presented as e.g. lines of: fields, contours, flows, stresses, strains and displacements (Jackson et al., 2008). Computer techniques also enable simulations of values changes influence of selected models parameters on obtained solutions. Thus obtained effects allow verifications of models when compared with results of experimental researches and facilitate evaluation of anticipated consequences of the models modifications in relation to real objects (Zienkiewicz & Taylor, 2000a, 2000b; Ziętkowski, 2007).

The method of finite elements is a numerical method of analysis applied for defining approximate solution in many engineering issues. The first mentions of application of the method of finite elements appeared at the beginning of the '60s last century when the method of approximation with the use of triangular elements with minimization of functional suggested by Courant in 1943 got some interest. As a results of works conducted by Zienkiewicz concerning possibilities of applying the method of finite elements for solving problems which can be described with the use of variation problems, its interpretation range has been significantly expanded since 1965.

2. Algorithm of application of the method of finite elements for analysis of concrete and rocks

Algorithm of application of the method of finite elements for analysis of concrete and rocks disintegrated by EHD method consisted of the following steps (Zienkiewicz & Taylor, 2000a, 2000b):

- Integral assignment I (functional) in a variation problem to boundary conditions, resulting from boundary Γ limiting the region Ω, in which the solution is sought (function φ minimizing the functional),
- Division of the region Ω onto sub-regions (finite elements),

- Approximation of integrand function φ with polynomials inside finite elements,
- Defining the function φ in the whole region Ω, minimizing the functional.

Functional is the name for integral I, which can be expressed with the formula (Zienkiewicz & Taylor, 2000a, 2000b):

$$I = \iint_{\Omega} F(u, u_x, u_y, x, y)dxdy \tag{1}$$

The integral defined for flat state for displacements (relative strains) describes combinations of some unknown functions $u(x,y)$ and their derivatives so that its value became extreme.

The principle of the method of finite elements regarding an examined system based on digitization and approximation of the continuous region occupied by the system with a determined number of finite elements. The elements got joined in a number of points, treated as nodal ones, which exist at boundaries and in particular cases also inside elements approximating the region. State of displacements was described within each element using simple functions which must meet the conditions of continuity in nodes. Next, minimizing the energy of the whole system, which can be presented as a function of nodes displacements, the value of displacements and reactions in free and immobilized nodes was determined. Knowing nodes displacements considering initial conditions and constitutive properties of materials the following states were defined: displacement and stresses inside and on elements boundaries. Next, there was defined a system of forces concentrated in nodes, which balance boundary stresses and applied outside forces, being the base of stiffness connections described in the equation (Zienkiewicz & Taylor, 2000a, 2000b; Ziętkowski, 2007):

$$\{F\}^e = [K]^e \{\delta\}^e + \{F\}_p^e + \{F\}_{\varepsilon o}^e \tag{2}$$

where:

$\{F\}^e$ — forces concentrated in nodes
$[K]^e$ — stiffness matrix.
$\{\delta\}^e$ — set of nodes displacements for a given element.
$\{F\}_p^e$ — node forces essential for balance of each external load affecting the element.
$\{F\}_{\varepsilon o}^e$ — node forces induces by initial strains.
e — subsequent number of a node in finite element

As it was previously mentioned the region in which the analysis by the method of finite elements is conducted was covered with digitized network of finite elements. In order to fully represent the shape of the analyzed surface or volume, there were efforts undertaken to perform its possibly most accurate approximation. Using two-dimensional field variable $\varphi(x,y)$, a segment approximation of the limited region was done, indicating that node values clearly define the variable on the whole concerned region in the plane *xy*.

If we accept that region Ω of a boundary Γ presented in Fig. 1 was divided onto triangular elements with nodes on the vertices of the triangle, the type of region digitization leads to linear dependency $\varphi(x,y)$ within the boundaries of each element presented in Fig. 2.

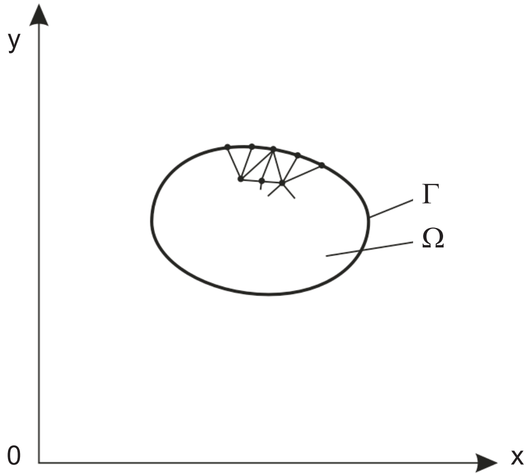

Fig. 1. Division of two-dimensional region Ω onto triangular elements (Zienkiewicz & Taylor, 2000a, 2000b)

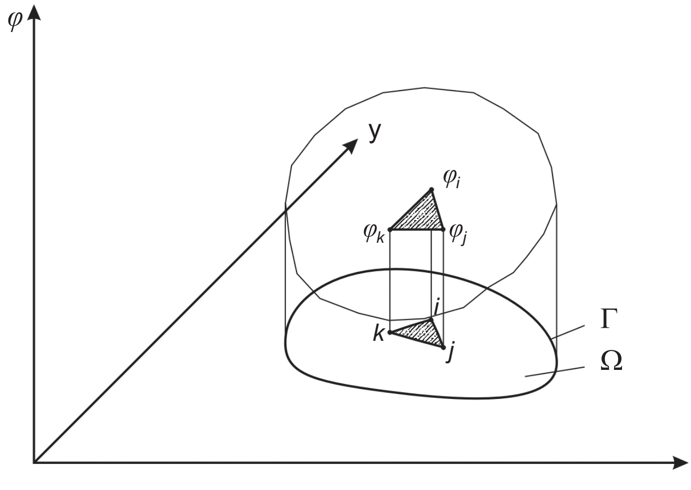

Fig. 2. Division of region Ω with a fragment of linear solution (Zienkiewicz & Taylor, 2000a, 2000b)

The plane crossing three node values connected with element *(e)* is described by the following equation:

$$\varphi^{(e)} = \beta_1 + \beta_2 x + \beta_3 y \tag{3}$$

The constants β_1,β_2,β_3 can be expressed by node coordinates of *e*-tuple element and node values φ, using in the expression (3) values of particular nodes coordinates. As a result the following system of equations is obtained:

$$\begin{cases} \varphi_i^{(e)} = \beta_1 + \beta_2 x_i + \beta_3 y_i \\ \varphi_j^{(e)} = \beta_1 + \beta_2 x_j + \beta_3 y_j \\ \varphi_k^{(e)} = \beta_1 + \beta_2 x_k + \beta_3 y_k \end{cases} \tag{4}$$

The solution of the system of equations (4) is as follows:

$$\begin{cases} \beta_1 = \dfrac{\varphi_i (x_j\, y_k - x_k\, y_j) + \varphi_j (x_k\, y_i - x_i\, y_k) + \varphi_k (x_i\, y_j - x_j\, y_i)}{2\Delta} \\ \beta_2 = \dfrac{\varphi_i (y_j - y_k) + \varphi_j (y_k - y_i) + \varphi_k (y_i - y_j)}{2\Delta} \\ \beta_2 = \dfrac{\varphi_i (x_k - x_j) + \varphi_j (x_i - x_k) + \varphi_k (x_j - x_i)}{2\Delta} \end{cases} \tag{5}$$

where:

$$2\Delta = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}$$

Δ — area of triangular element surface of nodes i,j,k.

If we introduce values of constants described by the system of equations (5) to the equation (3) describing the plane crossing three node points, the following plane equation is achieved:

$$\varphi^{(e)} = \frac{a_i + b_i x + c_i y}{2\Delta} \varphi_i + \frac{a_j + b_j x + c_j y}{2\Delta} \varphi_j + \frac{a_k + b_k x + c_k y}{2\Delta} \varphi_k \tag{6}$$

where:

$a_i = x_j y_k - x_k y_j$ $\quad$ $a_j = x_k y_i - x_i y_k$ $\quad$ $a_k = x_i y_j - x_j y_i$

$b_i = y_j - y_k$ $\quad$ $b_j = y_k - y_i$ $\quad$ $b_k = y_i - y_j$

$c_i = x_k - x_j$ $\quad$ $c_j = x_i - x_k$ $\quad$ $c_k = x_j - x_i$

After introduction of denotations:

$$N_n = \frac{a_n + b_n x + c_n y}{2\Delta} \qquad n = i, j, k \tag{7}$$

and

$$\varphi^{(\varepsilon)} = \begin{bmatrix} \varphi_i \\ \varphi_j \\ \varphi_k \end{bmatrix}, \qquad \boldsymbol{N} = \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix} \tag{8}$$

The equation (1.6) can be presented in the following form:

$$\varphi^{(e)} = \boldsymbol{N}^{\boldsymbol{T}} \varphi^{(\varepsilon)} = N_i \varphi_i + N_j \varphi_j + N_k \varphi_k \tag{9}$$

Functions N are called shape functions, basic functions or interpolation functions for a three-node triangular element. Knowing the node values φ we are able to present the whole solution area $\varphi(x,y)$ as a series of connected planes defined by shapes of the finite element applied in the analysis. Such a plane does not have discontinuities or slits between elements boundaries because potential value in each of the two nodes defining the element boundary clearly determines its linear change along the boundary.

The procedure of forming equations for a given element, and then creation of systems of equations for all elements covering the whole region result from the assumption that basic functions N meet certain conditions.

According to mathematical conditions of convergence the method of densification of element network used to increase calculations accuracy should consider following requirements:

- The elements must be diminished in such a way that each point of the region in concern was inside the element regardless of its size,
- All former networks must by comprised in the concentrated network,
- The form of basic functions must remained unchanged during the process of network densification.

It should be noticed that when elements with rectilinear boundaries are used for modelling a region with curvilinear boundaries, the first two conditions are not met and the convergence condition cannot be met. Despite the limitations, many applications of the method of finite elements for problems in regions not being polygons lead to sufficiently good engineering solutions. In order to ensure the monotonic convergence and to collect equation of particular elements, basic functions N must meet two conditions:

- Compatibility stating that on the element boundaries the field variable φ and each of its derivatives up to the grade smaller by one than the highest grade of the derivatives appearing in the functional $I(\varphi)$ must be continuous.

– Being complete where the unknown and its partial derivatives up to the highest grade occurring in the functional $I(\varphi)$ should be represented in $\varphi^{(e)}$ when at the boundary the element measurements approach zero (Zienkiewicz & Taylor, 2000a, 2000b).

3. Numerical analysis of the process of concrete and rock disintegration with the use of electrohydraulic effect

In the conducted analysis of concrete and rocks disintegration by electrohydraulic method there was applied MES software in the form of MSC.Marc/Mentat pack. The formerly described equations (1) to (9) got implemented into the pack. It was assumed that the calculations would be conducted for tests in the form of a cubic block of dimensions 1.0 × 1.0 × 1.0 m, whose model made in the MSC. Marc/Mentat pack was presented in Fig. 3. In the block a shot hole of diameter φ 38 mm was modelled at depth corresponding to 1/3 of the height of the disintegrated block. In the whole volume of the analysed block a network of finite elements of the shape of four-node tetrahedrons

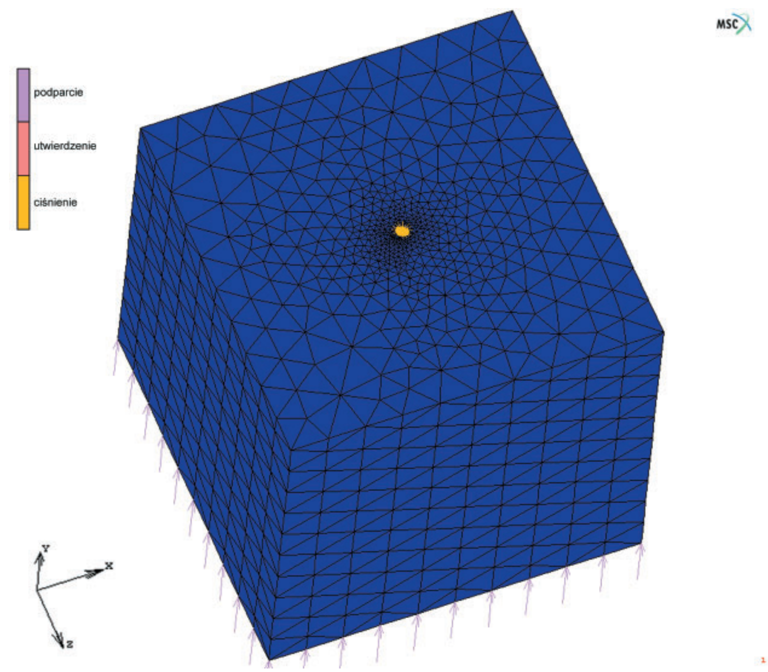

Fig. 3. Model of concrete block modelled in MSC.Marc/Mentatpack, disintegrated by electrohydraulic method (EHD) with marked boundary conditions

with nodes on elements vertices. In order to more accurate determination of the moment of the first cracks appearance, the network around the shot hole got concentrated (Ziętkowski, 2007).

Several simplifying assumptions were accepted to describe complex mechanical phenomena occurring during disintegration of a rock block by electrohydraulic method:

- Propagation of pressure generated by electrohydraulic effect is rectilinear and its value increases linearly in particular node points,
- During electrohydraulic discharge there are no pressure losses at the shot hole outlet,
- The material which the disintegrated block is made from is of a homogenous structure (isotropic),
- The block rests freely on the basic plan, is affected only by gravitational force, there are no primary stresses and initial loads,
- As required physic-mechanical parameters of disintegrated rock blocks there were accepted their compressing endurance R_c, thermal conductivity p_c and specific density ρ_o.

The Crack Data option included in the MSC.Marc/Mentat pack allowed prediction of the moment of materials crack initiation and simulation of the endurance fall connected with the phenomena. At the simulation for concrete blocks there were used models existing in the pack, whereas for blocks made from natural rocks some unique models were worked out by the authors. Applied analytical procedures precisely determining states of strains and stresses in constructions made of materials of low endurance parameters (e.g. concrete, rocks) are simultaneously complicated by different factors like:

- Low endurance of brittle materials resulting in occurrence of progressing cracks during the load increase,
- Nonlinear response of the load system – strain during multidimensional compressing of brittle materials.

On the basis of analysis conducted with the use of implemented or worked out models of concrete or rock blocks it is possible to obtain generalised distributions of strains and displacements for the disintegrated block as well as distributions of strains connected with occurrence of first cracks in the considered block. Distribution of strains is particularly helpful for determining the place of first cracks and the further progress of block disintegration (Ziętkowski, 2007).

The simulation of rock block disintegration by electrohydraulic effect was conducted for four concrete blocks made from concrete of class B7, B15, B25 and B35 successively and for rock blocks made from sandstone, dolomite and granite. It was assumed that the value of pressure would increase from 0 to the value exceeding the block endurance against uniaxial compressing. Calculations would be stopped at the moment of first cracks occurrence within the shot hole.

For the analysed concrete blocks there was accepted an endurance value against uniaxial compressing R_c equal to guaranteed endurance i.e. for concrete of class B7 – R_c = 7 MPa, for concrete of class B15 – R_c = 15 MPa, for concrete of class B25 – R_c = 25 MPa and for concrete of class B35 – R_c = 35 MPa. Thermal conductivity p_c for all blocks amounted at 0.16 W/m K, whereas specific density ρ_o = 1.6 kg/dm^3. For blocks made from natural rocks the mean values of: endurance against uniaxial compressing, thermal conductivity and specific density were accepted as follows, respectively: for sandstone – R_c = 85 MPa, p_c = 0.18 W/m K and ρ_o = 2.55 kg/dm^3, for dolomite – R_c = 130 MPa, p_c = 0.18 W/m K and ρ_o = 2.6 kg/dm^3, for granite – R_c = 215 MPa, p_c = 0.18 W/m K and ρ_o = 2.7 kg/dm^3.

In Fig. 4 there was presented a generalised (according to Huber-Mises hypothesis) distribution of strains induced in the first analysed concrete block, made from concrete of class B7 as a result of influence of pressure created in the shot hole. Concentric layout of strains distribution coincides with the shock wave moving in the material created as a result if the discharge. Generalised distribution of displacements for the above mentioned block and for the same moment of the simulation got presented in Fig. 5.

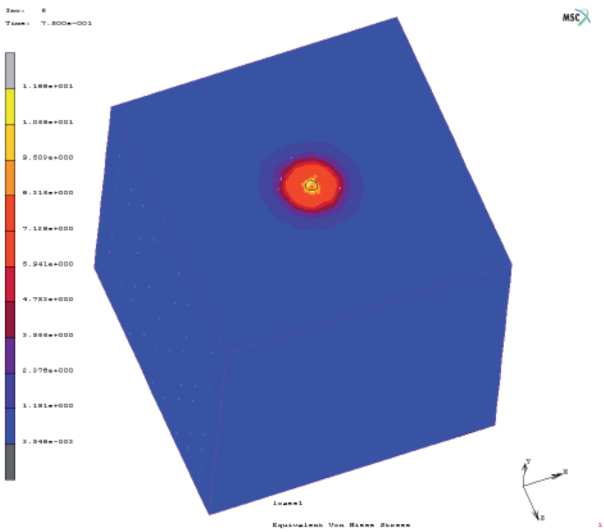

Fig. 4. Generalised distribution of strains for a concrete block

The most valuable information obtained during the simulation of rock block disintegration by electrohydraulic effect is gaining distribution of strains connected with the occurrence of the first cracks. In Figs. 6÷12 there were presented distributions of strains connected with occurring first cracks in disintegrated blocks made from four selected

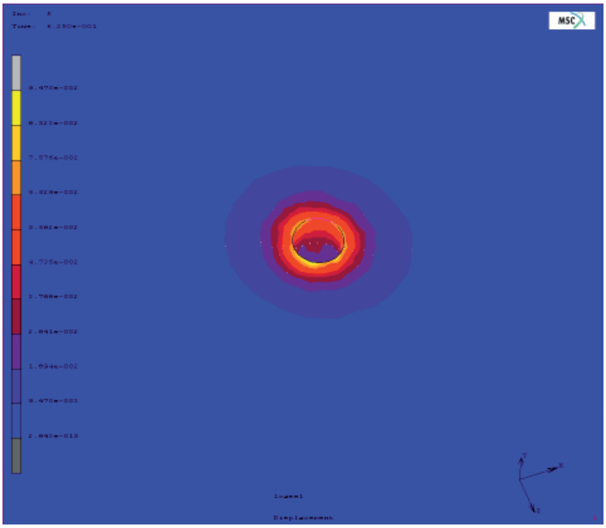

Fig. 5. Generalised distribution of displacements for a concrete block

types of concrete: class B7, B15, B25 and B35 and natural rocks – granite, dolomite and sandstone.

As it can be observed in most cases the cracks appear along lines perpendicular to side boundaries of the block, going through the middle of the shot hole. According

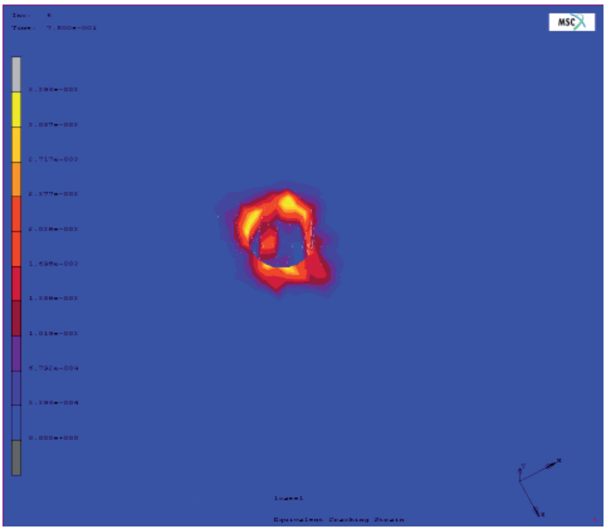

Fig. 6. Distribution of strains connected with occurrence of the first cracks for the block made of concrete class B7

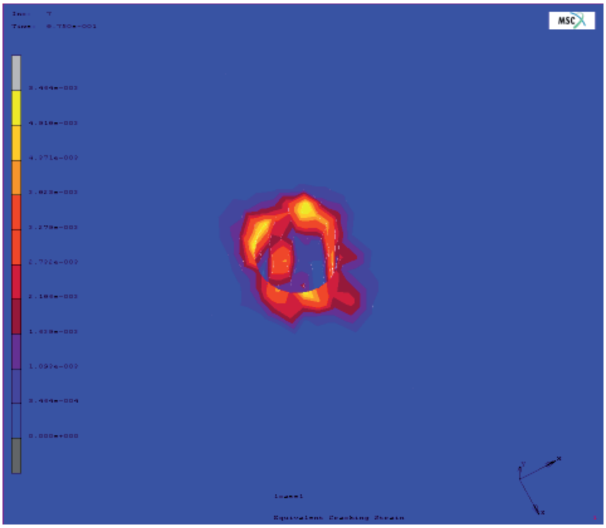

Fig. 7. Distribution of strains connected with occurrence of the first cracks for the block made of concrete class B15

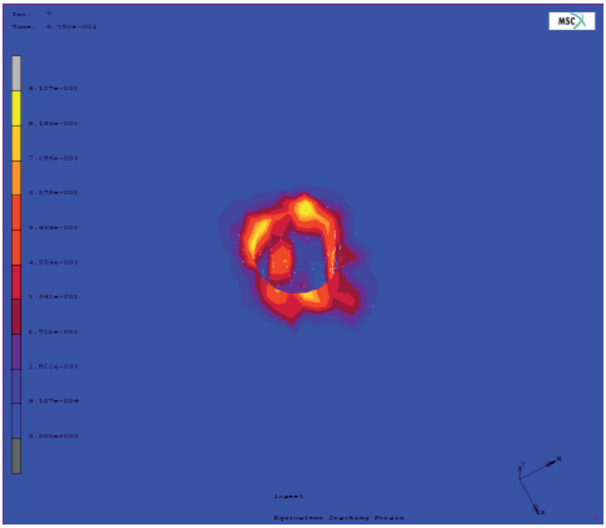

Fig. 8. Distribution of strains connected with occurrence of the first cracks for the block made of concrete class B25

to observations conducted during previously performed tests of disintegration of rock blocks, the lines cross each other at angles close to 90 degrees. It is particularly visible in Fig. 10 presenting analysis results for the sandstone block.

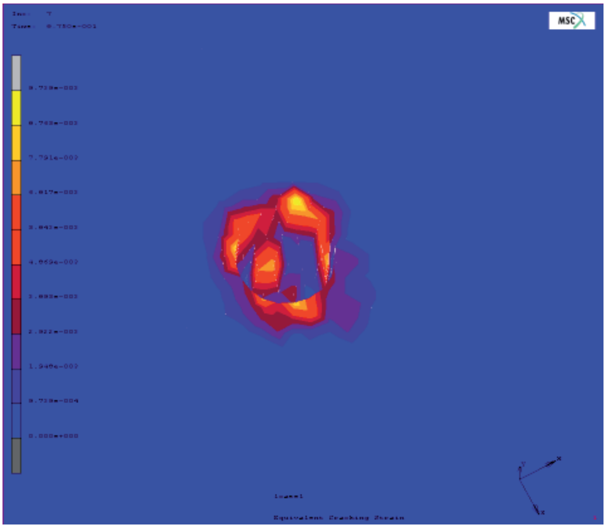

Fig. 9. Distribution of strains connected with occurrence of the first cracks for the block made of concrete class B35

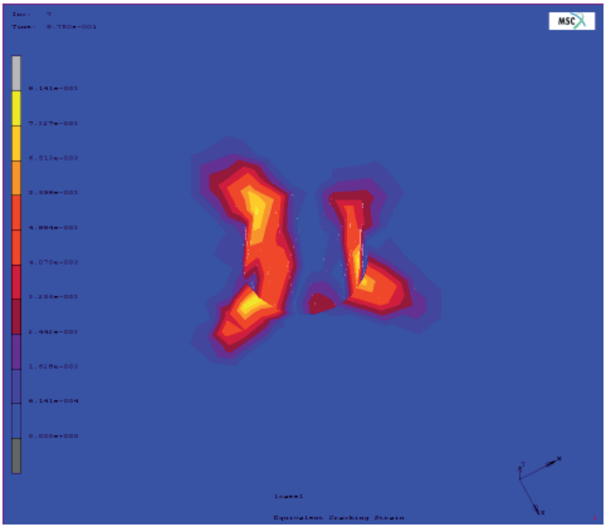

Fig. 10. Distribution of strains connected with occurrence of the first cracks for the sandstone block

The numerical analysis conducted within the chapter also allowed determination of minimal values of pressure affecting the shot hole at which the first cracks appear in the system. The values got presented in Table 1.

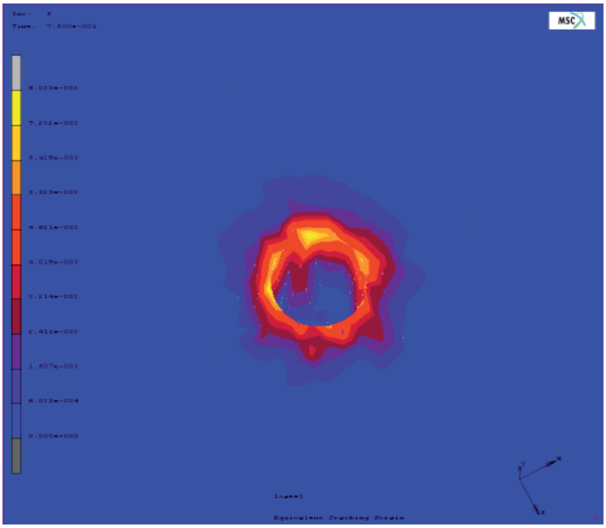

Fig. 11. Distribution of strains connected with occurrence of the first cracks for the dolomite block

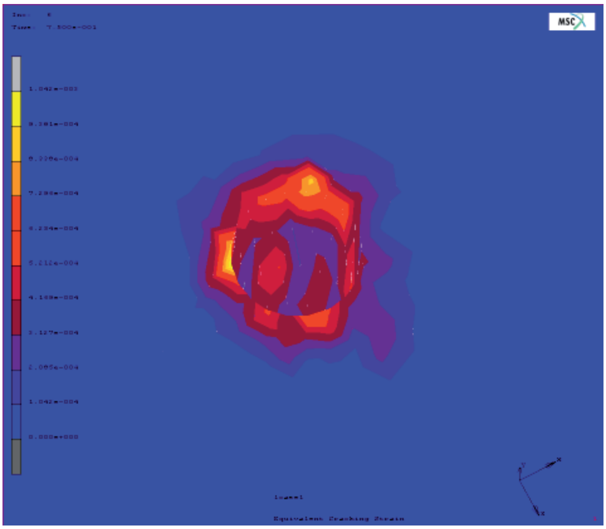

Fig. 12. Distribution of strains connected with occurrence of the first cracks for the granite block

TABLE 1

Values of pressure in the shot hole at which the process of cracking begins

Block material		Pressure [MPa]
Concrete	B 7	17
	B 15	34
	B 25	56
	B 35	75
Granite		35
Dolomite		15
Sandstone		11

4. Experimental confirmation of MES analysis

In chapter 2 there were introduced analytically defined distributions of strains affecting blocks made from rocks and concrete and structures of cracking of the blocks when treated with the use of the electrohydraulic method. The obtained results are an effect of solving mathematical models implemented in the calculation pack based on the method of finite elements in which there were used several simplifying assumptions for description of complex mechanical phenomena occurring during the treatment. For the verification of the analytically obtained results, a cycle of experimental researches got undertaken in order to (Erlay & Martin, 1966; Neville, 2004):

- Obtain a real distribution of cracks in the examined blocks,
- Perform a set of tests of concrete blocks disintegration by the electrohydraulic method.

Assuming that one of the most significant features of concrete, deciding about its susceptibility to disintegration with the use of the electrohydraulic effect is its endurance against compressing, the tests included blocks made from concrete of the following classes:

1. B7 – compressing endurance 7 MPa,
2. B15 – compressing endurance 15 MPa,
3. B25 – compressing endurance 25 MPa,
4. B35 – compressing endurance 35 MPa,

The W/C indicator of the above mentioned concretes was presented in Table 2.

TABLE 2

W/C indicator for tested concrets

Concrete type	B7	B15	B25	B35
Compressing endurance [MPa]	7	15	25	35
W/C indicator	0.85	0.65	0.48	0.39

For the needs of the tests a samples were made of dimensions $0.85 \times 0.85 \times 0.85$ m, which stated the volume of 0.614 m^3, and of dimensions $1.0 \times 1.0 \times 1.0$ m, of the volume = $1 m^3$, and the accepted periods of concrete curing were as follows: 7, 14, 21 and 28 days.

Analysing the previously introduced physic-mechanical properties of concretes, the following qualities were decided to be the most representative (Neville, 2004):

- Compressing endurance,
- Stretching endurance,
- W/C indicator,
- Concrete curing time,
- Conditions of curing.

On the basis of experience and results obtained by other authors it was accepted during the tests that optimal depth of the shot hole is the one amounting at 30-40% of the height of the disintegrated block .

The charging voltage of pulse condensers which means the energy of impulse wave was thought to be the basic parameter influencing the disintegration effects (Erlay & Martin, 1966; Neville, 2004).

The disintegrated samples were of the volume from 0.216 to 1.0 m^3 and were made from concretes: B7, B15, B25 and B35. All samples were cured in dry air for the period of 28 days so until they reached full endurance (tab. 3).

TABLE 3

Disintegration of concrete samples made from concretes: B7, B15, B25 and B35

Sample No.	Sample dimensions[m]	Sample volume [m^3]	Concrete type by compressing endurance [MPa]	Voltage essential for disintegration [kV]	Eenergy of impulse wave [kJ]
1	$0.6 \times 0.6 \times 0.6$	0.216	7	5.8	18.5
2			15	5.5	16.6
3			25	5.4	16.0
4			35	5.1	14.3
1	$0.8 \times 0.8 \times 0.8$	0.512	7	6.4	22.5
2			15	6.2	21.4
3			25	6.0	19.8
4			35	5.9	19.1
1	$1.0 \times 1.0 \times 1.0$	1.0	7	8.0	35.2
2			15	7.7	32.6
3			25	7.5	30.9
4			35	7.3	29.3

The confirmation of theoretical studies was sought through experiments, conducting several tests of disintegration of concrete and rock blocks. The effects of the tests partially confirmed the theory. It is obvious that in case of such heterogeneous material like rock it would be naive to expect a 100% confirmation of theory through experiments. Nevertheless, the conducted reasoning and the tests indicate on the author's opinion the fact that in some certain cases and for some most homogeneous rocks the method of finite elements can be useful for defining rocks reactions when undergoing violent waves effects causing disintegration of their structure.

An example of disintegration of a concrete block presented below is to confirm the statements (Fig. 13).

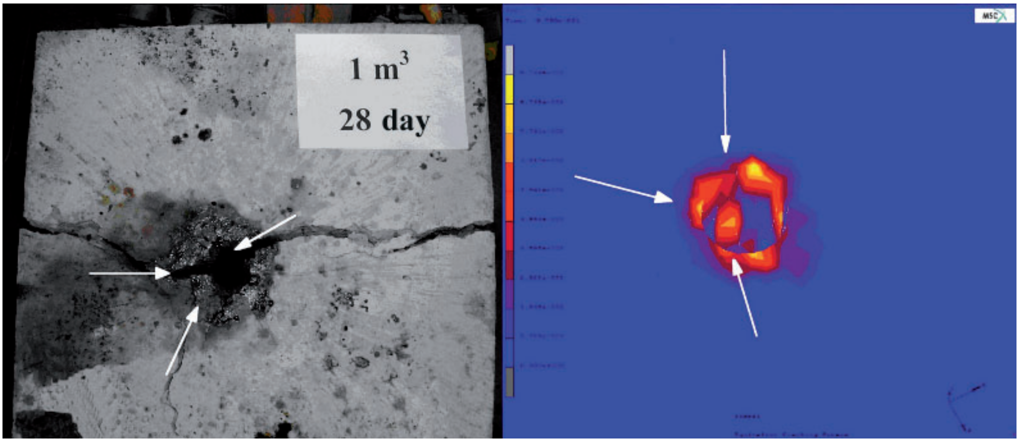

Fig. 13. Comparison of disintegrated concrete block (volume 1.0 m^3, 28-day concrete) with the result of MES simulation

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